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Use of a Particle Size Distribution Model to Predict Unsaturated Hydraulic Conductivity of Soils of Andaman and Nicobar Islands

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ABSTRACT

The present study deals with the prediction of hydraulic conductivity, *K*, as a function of water content, θ, of 9 soils of Andaman & Nicobar Islands, India, 3 each in clay loam, sandy clay and clay textures, from the particle-size distribution (PSD) data, using Arya-Paris model. The pore-size distribution was derived from PSD data using the model and the experimental $K(\theta)$ was determined by the horizontal infiltration method. Nine soils, 3 each with the above-mentioned textural classes were used to relate the pore flow rate (q) and the pore radius (r) using the parameters c and x, as obtained from the Hagen-Poiseuille equation for an idealized porous medium. For the textural classes under study, $\log c$ varied from -4.857 to 0.147 and x varied from 3.216 to 4.487, but no systematic trends could be followed. The model predicts unsaturated hydraulic conductivity with reasonable accuracy. The root mean square residuals (RMSR) of the log-transformed $K(\theta)$ for all textures ranged from 0.107 to 0.879. The intra and inter-textural uncertainties in the prediction could be attributed to the heterogeneity in the experimental data, which is originated from the difference in hydrophysical behaviour of the studied soils.

Key words: Particle-size distribution, pore-size, hydraulic conductivity.

Introduction

Hydraulic conductivity $K(\theta)$ is one of most variable soil physical parameters, direct estimation of which may have practical limitations due to many factors including detailed procedures, time-consumption and difficulty in operation (van Genuchten and Leji, 1992; Shao and Horton, 1998; Chaudhari, 1998). This has led to the efforts of estimating $K(\theta)$ from easily available data like particle-size distribution (PSD), bulk density, organic carbon of soil *etc.* (Brooks and Corey, 1964; Mualem, 1976; van Genuchten 1980; Bouma and van Lanen, 1987; Kool and Parker, 1988; Tyler and Wheatcraft, 1989, 1990; Arya and Dierolf, 1992; Wu and Vomocil, 1992; Woosten *et al.* 1995). These indirect ways have got several advantages over direct methods (van Genuchten and Leji, 1992; van Dam *et al.*, 1992) and thus gaining considerable attention these days (Mualem, 1992; Arya *et al.* 1999a).

Hydraulic conductivity models based on pore-size distribution of soils require soil water function $\psi(\theta)$ and the saturated hydraulic conductivity K_s as the two most important input parameters (Marshal, 1958; Mualem, 1976; van Genuchten, 1980; Arya *et al.* 1999a). Since pore-size distribution is also related to particle-size distribution (PSD), $\psi(\theta)$ could be quantitatively derived from PSD data with considerable accuracy (Arya and Paris, 1981; Tyler and Wheatcraft, 1989; Arya *et al.* 1999a; Arya *et al.* 1999b). This is significant since PSD data are readily available and can be easily determined in the laboratory. No significant information on $K(\theta)$ in Andaman & Nicobar Islands of India is available and no work has been done to either measure or predict the $K(\theta)$ functions of these soils. Thus, an attempt was made to predict the $K(\theta)$ in a watershed of Andaman & Nicobar Islands based on one of the well-accepted models proposed by Arya and Paris (1981).

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Materials and Methods

Undisturbed soil core samples were collected from the watershed adjoining the Central Agricultural Research Institute, Port Blair, Andaman and Nicobar Islands. The PSD analysis was carried out using International Pipette method after passing the soils through 2 mm sieve (Gee and Bauder, 1986) and the textural classification was accomplished following International Society of Soil Science (ISSS) scheme. Moisture retention characteristics of the soils were determined using pressure plate/ membrane apparatus at 33, 60, 100, 500, 1000 and 1500 KPa suction pressures (Richards, 1965). Soil water diffusivity $D(\theta_x)$ was calculated from experimental water content profile of undisturbed soil columns using the method described by Bruce and Klute (1965). Unsaturated hydraulic conductivity $K(\theta_x)$ was calculated by using the relationship:

$$K(\theta_x) = D(\theta_x) * C(\theta_x)$$

where, $C(\theta_x)$ is the specific water capacity of soil at $\theta = \theta_x$ and expressed as the inverse of slope of the experimental soil moisture retention curve.

The Model

The Arya-Paris model (Arya and Paris, 1981; Arya *et al.* 1999b) is based on the principle that flow in soil pores is a function of pore radius with assumptions that only the completely filled pores contribute to the hydraulic conductivity, at a given saturation with negligible contribution from the partially drained pores. If the particle size distribution curve is divided into number of size fractions, the solid mass in each fraction can be assembled to form a hypothetical, close-structure with uniform size spherical particles. The flow rate of an individual pore (considering it as a capillary tube) can be calculated based on Hagen-Poiseuille law for capillary flow, as:

$$q_i = cr_i^x \tag{1}$$

where, r_i is the mean pore radius (cm) for *i*th size fraction, c and x are the parameters of the model. r_i can be obtained from a relation between mean particle radius for the *i*th particle size fraction (cm), void ratio, the number of equivalent spherical particles in the *i*th fraction and a scaling parameter a*i*; Parameter *c* takes into account of the complexity involved in accurately describing the pore shape (expressed as a shape parameter *S*) and difficulty in incorporating other factors that influence flow rates through soil pores (Arya *et al.* 1999b).

Parameters *c* and *x* can be evaluated from plots of $\log(q_i)_m$ vs. $\log r_i$ data, where $(q_i)_m$ is the measured pore flow rate, which can be obtained from the measured values of hydraulic conductivity $K(\theta_i)_m$ corresponding to the calculated water contents θ_i in the experimental soil moisture characteristic curve.

The scaling parameter α_i (Arya *et al.* 1999a) is defined as $\alpha_i = \log N_i / \log n_i$, where, n_i is the number of spherical particles in the i^{th} particle size fraction (obtained by the solid mass fraction w_i and mean particle radius R_i) and N_i represents the number of spherical particles of radius R_i required to trace the tortuous pore length contributed by the same solid mass in a natural structure soil matrix.

Finally, the hydraulic conductivity as a function of water under a unit hydraulic gradient can be calculated using the following relation:

$$K(\theta_i) = c\phi / \pi \sum_{j=i}^{j=i} R_j^{(x-2)} w_j [0.667\, en_j^{(1-\alpha)}]^{(x-2)/2} \tag{2}$$

Evaluation of model parameters c and x

The parameters *c* and *x* were calibrated by fitting experimental hydraulic conductivity data to the flow model given by Eq. 1 and followed by the procedure explained by Arya *et al.* (1999a and b) for each of the three textural classes. The evaluated values of *c* and *x* were used to predict the *K(θ)* functions from the PSD data. In the process, 9 soils, 3 each with clay loam, sandy clay and clay texture, and 6 soils, 2 each with the same textural classes, were used for calibration and testing of the model, respectively. The values of model parameters *x* and *c* for the experimental soils used for its calibration along with R^2 log of shape factor S and the root mean square are presented in Table 1.

Results and Discussion

A linear relationship was obtained between log of experimental pore flow rate, q_i, and log of pore

Table 1. Three Indian soils used for model calibration, model parameters c and x, R^2 and the log of shape factor (S)

Texture	Location code	Sand	Silt	Clay	ρ_b Mg m^{-3}	log c	x	R^2	log S
		————	(%)	————					
Clay loam	B1	39.3	18.0	42.7	1.38	-4.520	3.432	0.924	10.0587
	D3	45.1	18.0	36.9	1.27	-3.947	3.636	0.911	9.4857
	E2	43.6	16.4	40.0	1.30	-4.857	3.216	0.959	10.398
Sandy clay	B2	45.3	19.5	35.2	1.25	-2.496	4.105	0.903	8.0349
	C2	49.9	14.0	36.1	1.25	-2.378	4.071	0.940	7.9164
	F1	46.5	17.5	36.0	1.30	0.147	4.487	0.937	5.3918
Clay	B3	46.3	14.2	39.5	1.15	-0.419	3.331	0.926	5.9579
	K1	40.5	19.4	40.1	1.21	-0.588	3.484	0.948	6.1264
	K2	37.4	18.3	44.3	1.24	-0.057	3.679	0.967	5.5952

radius, r_i, for 9 Indian soils representing clay loam, sandy clay and clay textures (Fig.1). The Hagen-Poiseuille pore flow rates were calculated assuming $x=4$ and $S=8$ (for cylindrical tube of uniform diameter). The viscosity of water was taken as 0.001 poise. The experimental pore flow rates were calculated from measured values of $K(\theta)$ as proposed by Arya *et al.* (1999b). Similarly, logqi for individual textural class was found to have good linear relationship against logri of the respective textural class. The coefficients of determination (R^2) values were found as 0.911, 0.924, 0.959; 0.903, 0.937, 0.940; and 0.926. 0.948, 0.967, for clay loam, sandy clay and clay soils, respectively, collected from three different sites (Table 1). When all the textures pooled together, the R^2 value was recorded at 0.812 (Fig.1). The calculated values of log c, x and S are summarized in Table 1. Parameter x of

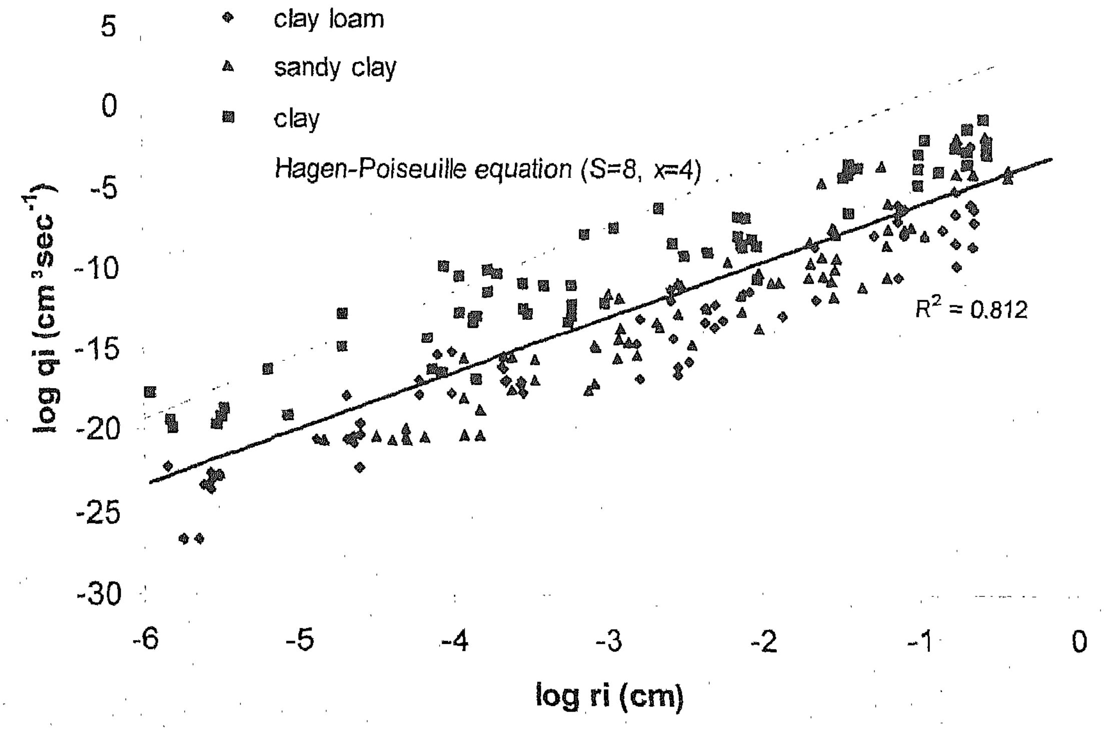

Fig. 1. Relationship between log of pore flow rate (q_i) and pore radius (r_i) for nine soils of different textural classes

the flow model (*i.e.* the coefficient of regression of the straight line equation relating logq_i and logr_i) was found to vary from 3.216 to 4.487. The values of x for soils were always found lower than the x in Hagen-Poiseuille equation. The values of log$c$ were observed to be negative for most of the textural classes and ranged between -4.857 and 0.147. The values of S were calculated from the $\log c = \log (\pi \rho_w g/S\eta)$ relationship, assuming viscosity of water at 25^0C as the other experimental data were obtained at the room temperature (Arya *et al.* 1999b). The range of values of S for different soil textures was wide and larger than the Hagen-Poiseuille equation, where $S = 8$. This wide variation is well in agreement with the similar findings by Arya *et al.* (1999b) and Chaudhari and Batta (2003) and could be attributed to the non-uniform pore geometry and pore-size distribution of the soils. These properties are likely to vary from one texture to another and so from one sample to another sample in a same textural class. Nevertheless, the results indicate that macroscopic flow behaviour of soils can be predicted from the Hagen-Poiseuille model for flow in straight capillary tubes.

The estimated hydraulic conductivity functions, $K(\theta)$, independently for 9 soils, 3 each of clay loam, sandy clay and clay were compared with experimentally determined $K(\theta)$, using horizontal infiltration method. The experimental and predicted data for clay loam, sandy clay and clay are presented in Fig. 2. The shape and pattern of predicted $K(\theta)$ functions matched more or less with those of the measured data in the water content range for which measured data were available. Thus, it could be observed that the present model could predict hydraulic conductivity values over a wider range of soil moisture contents in both saturation and dry extremes compared to measured values.

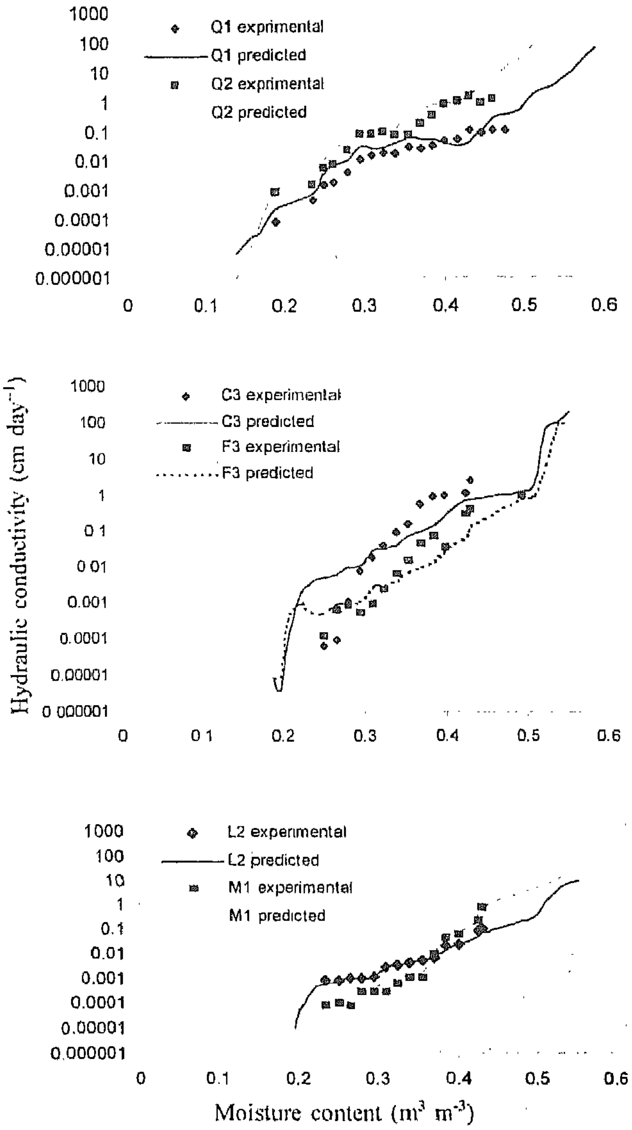

Fig. 2. Comparison of experimental and predicted unsaturated hydraulic conductivity of (a) clay loam (b) sandy clay and (c) clay soils

The comparison of the log of experimental vs. predicted values of hydraulic conductivity on a 1:1 scale is presented in Fig. 3. The regression lines between experimental and predicted $K(\theta)$ values could almost match the 1:1 line with R^2 of 0.950 for clay, 0.934 for clay loam, 0.894 for sandy clay soil. When the same is plotted for all soils the regression line is found to be again close to 1:1 line with R^2 value of 0.923. The RMSRs of the log-transformed predicted and experimental $K(\theta)$ found to be 0.334 and 0.378 for clay loam, 0.437 and 0.879 for sandy clay and 0.107 and 0.257 for clay (Table 2). The average root mean square residue (RMSR) for all textures was calculated as 0.523. The spread of data around 1:1 line was quite obvious in view of the difficulty and complexity involved in estimating hydraulic conductivity of soils. We also observed differences between measured hydraulic conductivity even the soil samples are of same texture class, which is in agreement with other workers (Dirksen, 1991; Arya *et al.* 1999b; Chaudhari and Batta, 2003). Variations

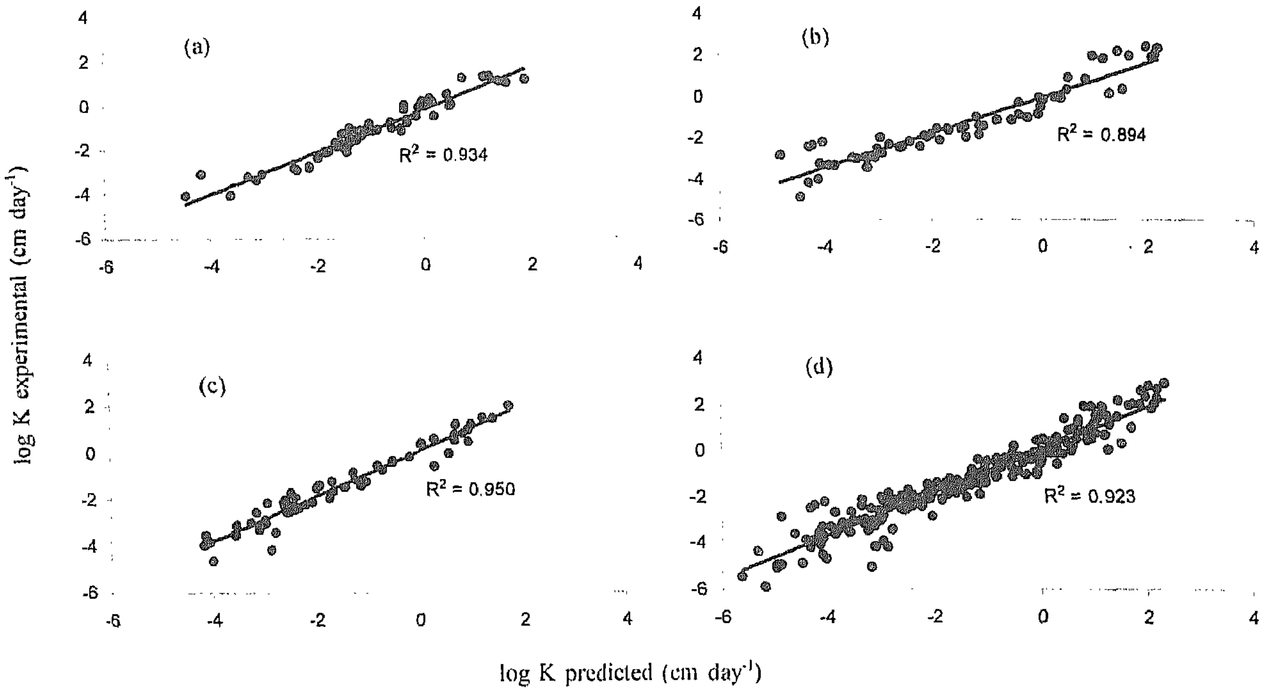

Fig. 3. Comparison of log-normal experimental and predicted unsaturated hydraulic conductivity of (a) clay loam, (b) sandy clay (c) clay and (d) pooled

Table 2. Three Indian soils used for model testing, root mean square residuals (RMSR) of log-transformed $K(\theta)_{predicted}$ and $K(\theta)_{mean}$ data

Texture	Location code	Sand	Silt	Clay	ρ_b	RMSR
			%		Mg m^{-3}	
Clay loam	Q1	42.2	18.7	39.1	1.31	0.334
	Q2	39.0	18.3	42.7	1.24	0.378
Sandy clay	C3	52.0	11.6	36.4	1.27	0.879
	F3	50.9	12.2	36.9	1.31	0.437
Clay	L2	40.2	18.4	41.4	1.35	0.107
	M1	43.7	14.4	41.9	1.33	0.257

between predicted and measured hydraulic conductivity are extensively reported in literature (El-Kadi, 1985; Mishra *et al.* 1989; Yates *et al.* 1992; Tamari *et al.* 1996; Poulsen *et al.* 1998). These variations emerge from the differences in PSD, bulk density, mineralogical compositions, structural properties and organic matter present in soils at the time of collection of samples and other physicochemical characteristics, even within soils of same textural class. In the present study, textural class average values of the model parameters *c* and *x* are used, which is likely to involve some errors in the prediction.

But nonetheless the predicted hydraulic conductivity of soils of the study area based on the model as proposed by Arya and Paris (1981) was in good agreement with the measured or true values of the same. However there are scopes of further improving the model by introducing factors that influence the flow processes in soil under unsaturated moisture regime.

Conclusions

The present study calibrates and tests a PSD-based model given by Arya and Paris (1981) to predict the unsaturated hydraulic conductivity of 9 soils of Andaman and Nicobar Islands representing three types of texture classes, clay loam, sandy clay and clay. The results show considerable success in predicting hydraulic conductivity from PSD data of these soils. The average values of the model parameters for a particular textural class were used in the study. As there are variations among the soil samples even within a specific textural class attributed to the differences in bulk density, organic matter content, and mineralogical composition *etc.*, textural similarities could not necessarily be translated into hydrophysical similarities. Some disagreement observed between the predicted and the measured data suggests that the model needs further improvement to include some more parameters, which influence the flow behaviour of the soils.

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